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C. G. A. Harrison, H. M. Carle and K. M. Creer

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Modelling the core magnetic field of the Earth

By C. G. A. Harrison and H. M. Carle
Rosenstiel School of Marine and Atmospheric Science, University of Miami,
4600 *Rickenbacker Causeway, Miami, Florida* 33149, *U.S.A.*

The magnetic field generated in the core of the Earth is often represented by spherical harmonics of the magnetic potential. It has been found from looking at the equations of spherical harmonics, and from studying the values of the spherical harmonic coefficients derived from data from Magsat, that this is an unsatisfactory way of representing the core field. Harmonics of high degree are characterized by generally shorter wavelength expressions on the surface of the Earth, but also contain very long wavelength features as well. Thus if it is thought that the higher degree harmonics are produced by magnetizations within the crust of the Earth, these magnetizations have to be capable of producing very long wavelength signals. Since it is impossible to produce very long wavelength signals of sufficient amplitude by using crustal magnetizations of reasonable intensity, the separation of core and crustal sources by using spherical harmonics is not ideal. We suggest that a better way is to use radial off-centre dipoles located within the core of the Earth. These have several advantages. Firstly, they can be thought of as modelling real physical current systems within the core of the Earth. Secondly, it can be shown that off-centred dipoles, if located deep within the core, are more effective at removing long wavelength signals of potential or field than can be achieved by using spherical harmonics. The disadvantage is that it is much more difficult to compute the positions and strengths of the off-centred dipole fields, and much less easy to manipulate their effects (such as upward and downward continuation). But we believe, along with Cox and Alldredge & Hurwitz, that the understanding that we might obtain of the Earth's magnetic field by using physically reasonable models rather than mathematically convenient models is very important. We discuss some of the radial dipole models that have been proposed for the non-dipole portion of the Earth's field to arrive at a model that agrees with observations of secular variation and excursions.

Introduction

The easiest way of describing observations of the magnetic field obtained over the surface of a sphere is to represent the magnetic potential as a sum of spherical harmonic functions of the form

$$V = a \sum_{n=1}^{n_{\max}} \sum_{m=0}^{n} (a/r)^{n+1} \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) P_n^m (\cos \theta), \tag{1}$$

where V is the magnetic potential, n is the degree and m the order of the harmonic, a is a reference radius (usually the radius of the Earth), r is the radius at which the observation takes place, θ is the colatitude, ϕ is the longitude, $P_n^m (\cos \theta)$ are the Schmidt semi-normalized associated Legendre polynomials, and g_n^m and h_n^m are the Gauss coefficients. Equation (1) assumes that there are no sources external to the radius a. If (1) is differentiated along spatial coordinates, it gives the magnitude of the field components along the coordinates. Thus if F_{N}, F_{E} and F_{V} are the north, east and vertically downward components of field, they can be expressed as follows:

$$F_{\mathrm{N}} = \frac{1}{r}\frac{\partial V}{\partial \theta}, \quad F_{\mathrm{E}} = -\frac{1}{r \sin \theta}\frac{\partial V}{\partial \phi}, \quad F_{\mathrm{V}} = \frac{\partial V}{\partial r}. \tag{2}$$

It is possible to express the power in each degree of harmonic by using the formula of Lowes (1966). He showed that the mean square value over the surface of the sphere of the field $\boldsymbol{H}$ produced by harmonics of a given degree n, called R_n, is given by the following expression:

$$R_n = (n+1) \sum_{m=0}^{n} \{(g_n^m)^2 + (h_n^m)^2\}. \tag{3}$$

Langel & Estes (1982) have produced the most accurate set of harmonics for the Earth's magnetic field, up to degree 23. They used data collected by Magsat on a series of quiet days during the lifetime of this satellite. If lg R_n from this model is plotted against n, it turns out that the values between $n = 2$ and $n = 13$ fall on a steeply sloping line of negative slope, whereas the values for $n > 13$ fall on another line with a very small negative slope. The value for $n = 1$ demonstrates that the dipole term of the field is considerably more important than any other.

Bullard (1967) correlated the maximum degree and order of a set of spherical harmonics with a short wavelength limit of what is seen in a two-dimensional portion of the Earth's surface, and showed that spherical harmonics truncated at degree and order n will have the same short-wavelength cutoff as a two-dimensional Fourier series of the form

$$\sum_{p=0}^{P} \sum_{q=0}^{Q} a_{pq} \cos (2\pi/L)\,(px+qy)$$

for which $(P^2+Q^2)^{\frac{1}{2}} \leqslant nL/c$, where L is the length of the side of the square area and c is the circumference of the Earth. It is necessary to separate fields into core and crustal portions for two reasons. If we wish to look at anomalous magnetizations within the crust of the Earth generated by remanent or induced magnetization of ferromagnetic particles below their Curie point, then it is necessary to remove the field generated within the core of the Earth, whereas if we are interested in looking for sources of the core field, it is conversely necessary to filter out that component generated within the crust. Since these components should have grossly different wavelengths, Bullard suggested that the cutoff in spherical harmonics should be chosen from the standpoint of the maximum or minimum wavelength that it is desired to have within the spherical harmonic expansion.

There are two problems with this method. The first is that we do not measure potential at the Earth's surface, but rather either a component of the field (such as the vertical component), or more commonly the magnitude of the total field vector. The second problem is that higher harmonics of the potential have within them very long wavelengths, so that the separation between long and short wavelengths is not perfect. We shall show that if total fields are being used the first problem means that wavelengths equivalent to $c/2n$ are contained within harmonics up to degree n, meaning that shorter wavelengths are produced. We shall also discuss the second problem at some length.

On looking at the first problem in more detail, if the observed field is in fact the scalar total field, then in order to generate this from the spherical harmonic (s.h.) potential, it is necessary to obtain the three spatial derivatives as in (2), and then to square and add them. The process of squaring effectively halves the minimum wavelength represented by the harmonics. The process of taking the square root to obtain the scalar field also adds complications to the process, with the result that we have sometimes used the square of the total scalar field to reduce this one complication.

We shall show that the second problem is essentially unsolvable if s.h. are used to describe the field. Rather, we suggest that the core field should be represented by spatially distinct sources within the core. Thus we follow Cox (1968), Alldredge & Hurwitz (1964) and others who have suggested that dipoles located within the core may offer greater insight into the mechanisms whereby the core field is generated. We show that the use of radial dipoles buried within the core to represent core surface current loops can lead to a better approximation to the longer wavelength portions of the field than does the s.h. representation of the magnetic potential.

Data from Iceland and other areas with good secular variation records can be used to determine something about the nature of the source regions of the non-dipole field. The Icelandic data are especially important in that Iceland is at a very high latitude, and thus serves to check various models of excursions and reversals that have been proposed. The Icelandic data show several differences from data from lower latitudes. Firstly, the frequency of virtual geomagnetic poles (v.g.ps) of low latitude is considerably greater here than elsewhere. Secondly, the longitude of v.g.ps of low latitude from Iceland appears to be random, compared with longitudes of low-latitude v.g.ps from lower latitude sites. In this latter case, when the v.g.ps are from records of reversals, the longitude appears to be confined to lying close to the longitude of the site. For these and other reasons, Dodson (1980) suggested that the non-dipole field could be modelled by radial dipoles located within the core, in which the probability of finding a dipole within a certain area of core surface increased towards the poles. This is another example of using non-central dipoles to model the non-dipole field.

Table 1. Amplitudes of Fourier coefficients (10^6 nT^2)

wavelength ... zonal harmonics	$c/0$	$c/1$	$c/2$	$c/3$	$c/4$	$c/5$	$c/6$
$g_1^0\,g_3^0$	129.7	—	365.2	—	120.1	—	—
$g_2^0\,g_3^0$	—	37.4	—	16.8	—	7.2	—
$g_3^0\,g_3^0$	10.0	—	9.1	—	5.0	—	2.3

Separation of the field into core and crustal components

Suppose we take the spherical harmonic representation of the first three zonal harmonics of the field by using (1). The result is

$$V = a[(a/r)^2\, g_1^0 \cos\theta + (a/r)^3\, g_2^0\, \tfrac{1}{2}(3\cos^2\theta - 1) + (a/r)^4\, g_3^0\, \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta)]. \qquad (4)$$

Equation (2) can now be used to determine any one component of the field by differentiation in the appropriate spatial direction. Of more use to us is to obtain an expression for the scalar field, as this is the quantity that is most commonly measured when magnetic field surveys are made. This requires squaring, adding and then taking the square root of the three spatial derivatives of (4). The process of taking the square root complicates matters when we try to predict what a simple Fourier transform of a linear profile would look like (Carle & Harrison 1982), and so we have decided to work with the squared scalar field. An expression for this is given below:

$$\begin{aligned} T^2 = {} & (a/r)^6\,(g_1^0)^2\,(1+3\cos^2\theta) + 12(a/r)^7\, g_1^0 g_2^0 \cos^3\theta + (a/r)^8\, g_1^0 g_3^0\,(25\cos^4\theta - 6\cos^2\theta - 3) \\ & + \tfrac{1}{4}(a/r)^8\,(g_2^0)^2\,(45\cos^4\theta - 18\cos^2\theta + 9) + (a/r)^9\, g_2^0 g_3^0\,(45\cos^5\theta - 30\cos^3\theta + 9\cos\theta) \\ & + \tfrac{1}{4}(a/r)^{10}\,(g_3^0)^2\,(175\cos^6\theta - 165\cos^4\theta + \cos^2\theta + 9). \end{aligned} \qquad (5)$$

It is convenient to express this result in terms of cosines of integral multiples of θ rather than in terms of powers of cosines of θ. The resulting expression is shown below. This has been calculated for the surface of the Earth, where $r = a$:

$$\begin{aligned} T^2 = {} & \tfrac{1}{2}(g_1^0)^2\,(5+3\cos 2\theta)+3g_1^0g_2^0(3\cos\theta+\cos 3\theta) \\ & +\tfrac{1}{32}(g_2^0)^2\,(135+108\cos 2\theta+45\cos 4\theta) \\ & +\tfrac{1}{8}g_1^0g_3^0(27+76\cos 2\theta+25\cos 4\theta) \\ & +\tfrac{1}{16}g_2^0g_3^0(234\cos\theta+105\cos 3\theta+45\cos 5\theta) \\ & +\tfrac{1}{128}(g_3^0)^2\,(778+705\cos 2\theta+390\cos 4\theta+175\cos 6\theta). \end{aligned} \qquad (6)$$

Just as an example, suppose that we imagine that the 'core' field is represented by the first two zonal harmonics, and that the crustal field is represented by the third harmonic. Removal of the 'core' field will then produce an expression consisting of the last three terms in (6). The resulting 'crustal' field will have terms in integral multiples of θ all the way from zero to six. If we now take a longitudinal profile of this zonal field and perform a Fourier analysis, each expression in the last three terms of (6) will give a discrete Fourier component whose wavelength is equal to the circumference of the Earth divided by the integral multiplier of θ. Thus there will be Fourier components with wavelengths all the way from $c/0$ to $c/6$. The values of the amplitudes of these terms are given in table 1, by using values of the zonal harmonics from a recent spherical harmonic analysis of the field recorded by Magsat. It is immediately obvious from this table that the terms resulting in the interaction of the first and third harmonics are the largest, and have wavelengths of $c/0$, $c/2$ and $c/4$. Other wavelengths are also important. When it is realized that the values shown in the table are in units of squared field, we can see that amplitude of the $c/5$ harmonic is about 24% that of the $c/4$ harmonic. The result of the interaction on an odd harmonic (g_3^0) with the dominant first degree harmonic (g_1^0) is to produce Fourier components in which the even harmonics predominate. It would perhaps be better to use a spherical harmonic model in which all harmonics (zonal, tesseral and sectoral) are included. But the problem is that to write these terms down in order to illustrate principles is extremely tedious and lengthy. Zmuda (1958) has written down the total field squared produced by all harmonics up to degree 2. The resulting expression consisted of 21 terms.

Obviously, the above example is not particularly realistic. A more realistic example would be to take a complete spherical harmonic representation of the magnetic potential, including core and crustal sources, and to work out total field minus core field for a series of profiles along great circles, using as a model for the core field that field generated by the first n degrees of spherical harmonic, with n about 13. Unfortunately, such a model does not exist. It would require about 1.6×10^5 coefficients to achieve a resolution of 100 km, which is still too large to show up many crustal anomalies (Alldredge 1981). However, we may get an inkling as to what might happen by looking at the degree 23 spherical harmonic analysis produced by Langel & Estes (1982) in which they estimated that 'core' sources are important up to degree 13 and that 'crustal' sources are important for harmonics of degree 15 or above. We shall consider the 'core' field to be caused by harmonics up to degree 13, and shall produce Fourier analyses of residual or anomalous fields along great circle paths. These Fourier spectra are shown in figure 1.

These Fourier spectra show a plateau of power running from the first harmonic to about the 23rd harmonic, after which the power decreases until round-off errors produce a spurious signal at about harmonic number 33. It is possible to predict what the power spectrum would be from a hypothetical profile generated from a model in which higher degree harmonics were present. It would be expected that the power in the first 24 harmonics would be higher, because each additional degree of spherical harmonic used in the model to describe the crustal field would add power in this region. But we would also obtain much more power in harmonics between 24 and the maximum degree of the spherical harmonic model, after which the power would fall off rapidly to low levels. Figure 1 emphasizes that the removal of low-degree spherical harmonics, considered to be a 'core' field, does not remove all power at long wavelengths from surface scalar fields.

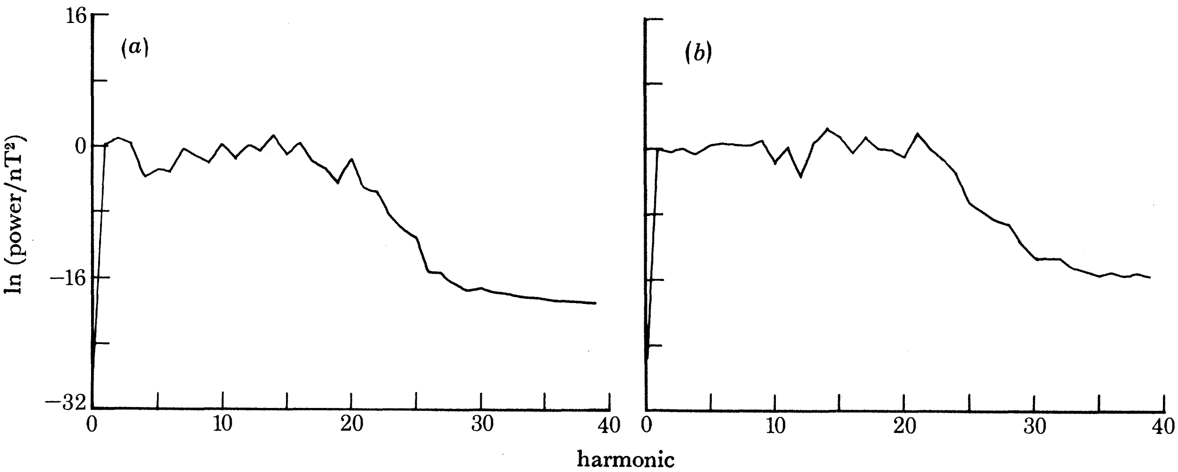

FIGURE 1. Fourier spectra of the residual field obtained by subtracting the scalar field generated by the first 13 degrees of spherical harmonic from the scalar field generated by the 23 degrees of spherical harmonic. The 23 degree harmonic field is that described by Langel & Estes (1982). (*a*) The profile is along the prime meridian with a fundamental length of the circumference of the Earth. (*b*) The profile is along the equator.

It is also possible to determine directly the magnetization necessary to produce the harmonics of potential thought to be caused by crustal magnetization. Chapman & Bartels (1940) have shown that there is a direct relation between the spherical harmonics of a vertically polarized thin magnetic shell and the spherical harmonics of its potential. The vertically magnetized shell is characterized by a vertical dipole moment per unit area. To obtain the Gauss coefficients for this function, it is necessary to multiply the g_n^m and h_n^m from (1) by the quantity

$$\frac{a}{4\pi}\left(\frac{2n+1}{n}\right)\left(\frac{a}{r}\right)^{n+1}.$$

Also, from Chapman & Bartels, the mean square value of $P_n^m(\cos\theta)\cos(m\phi)$ over the surface of the sphere is equal to $1/(2n+1)$. Therefore, from the values of R_n (equation (3)) given by Langel & Estes (1982) we can derive the mean square value of the vertical dipole moment per unit area:

$$\hat{M}^2 = \sum_{n=n_{\min}}^{n_{\max}} \frac{(2n+1)\,R_n}{(n+1)}\left(\frac{a}{4\pi n}\right)^2\left(\frac{a}{r}\right)^{n+1}\times 10^{-10}. \tag{7}$$

The factor of 10^{-10} is put in (7) because values of R_n are expressed in square nanoteslas. If the root mean square value of the dipole moment per unit area is divided by the thickness of the magnetized layer, we shall have obtained a root mean square value of the magnetization. We have summed the coefficients from $n_{min} = 14$ to $n_{max} = 23$ by using the data from Langel & Estes (1982) for an oceanic crustal layer 6 km thick, and obtained a magnetization of 1.05×10^{-3} e.m.u. (1.05 A m^{-1}) for the magnetization of an oceanic crustal layer. This magnetization is only for harmonics between 14 and 23. If the 'crustal' portion of lower degree harmonics (Langel & Estes 1982) and all the higher harmonics from 23 up to harmonics equivalent to the wavelength of the sea-floor spreading anomalies are included, a much higher magnetization would be obtained. Typical magnetizations of oceanic crustal rocks appear to be much too low for such magnetic potentials to be generated (Harrison 1976, 1981).

In these three examples (equations (6) and (7) and figure 1) we have shown that the 'core' field as generated by a certain number of low degrees of spherical harmonic does not completely remove long-wavelength terms from the surface scalar field. It is also possible to show that the same is true for field components, which follows simply from the higher degree terms in the associated Legendre polynomials. These all contain powers of cos θ going from powers of zero or one (depending on the degree and order of the harmonic) up to higher powers. We therefore suggest that a better model of the core field is to be found by modelling it with non-central sources, such as current loops located within the core.

Detailed analyses of this nature have been done by Alldredge & Hurwitz (1964) and Alldredge & Stearns (1969). Alldredge & Hurwitz (1964) found that to model the non-dipole field accurately, the off-centred dipoles needed to be placed deep within the Earth's core. The probable reason for this is that the dipoles are an attempt to model a source of finite dimensions at the core–mantle boundary.

The use of radial dipoles allows us to model the long-wavelength portion of the field better than the use of spherical harmonics. Each dipole can be modelled as a series of harmonics, which, if the origin of the spherical coordinate system is taken where the axis of the dipole impinges on the surface of the Earth, are all zonal harmonics. Alldredge (1980) has produced formulae for these harmonics. It is then possible to plot the mean square field generated by a dipole as a function of the degree of zonal harmonic, for different radial distances of the dipole from the centre of the Earth. Figure 2 shows three such plots. One is for dipoles located at the surface of the core. The other two are for dipoles placed at the maximum and minimum distances estimated by Alldredge & Hurwitz (1964). Also shown is the way in which the spherical harmonics of the Earth fall off, taken from Langel & Estes (1982). It can be seen that each dipole, if buried fairly deeply within the core, as required by the models of Alldredge & Hurwitz (1964) contributes harmonics which in general fall off more steeply than the spherical harmonic representation of the Earth's field.

Off-centred dipoles and the field in Iceland

Cox (1975) suggested that there was a certain pattern of non-dipole sources that explained some puzzling observations that he had made of excursions seen in lava flow data from Hawaii. These observations were that the excursions seemed always to be in the direction such that shallower inclinations of the field were produced. Cox suggested that this was caused by core–surface sources drifting past the longitude of Hawaii, which were preferentially oriented and occurred preferentially at certain latitudes. At low latitudes the core surface sources would be oriented such that the fields they produced would be pointing outward, whereas at 45° latitude the core surface sources would produce fields pointing downward. Cox pointed out that this arrangement would also produce a far-sided effect, which had been noticed by Wilson & Ade-Hall (1970), which could, however, also be explained by an off-centred dipole displaced towards the North Pole.

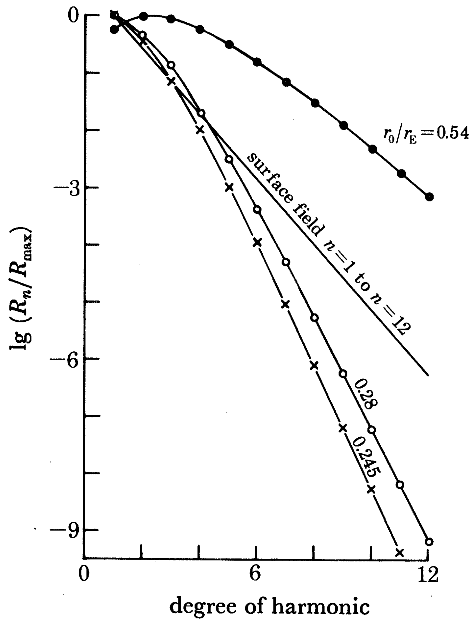

FIGURE 2. Normalized surface power as a function of degree of harmonic. See text for detailed description.

Unfortunately, the Icelandic data do not support this model. Harrison & Watkins showed (1979) that the zonal non-dipole field model of Cox should give a slightly near-sided pole at Iceland, whereas the data in fact suggest that even here there is a far-sided effect (Saemundsson *et al.* 1980). In addition, the model of Cox predicted that excursions in the Icelandic data should be marked by directions that have a higher inclination than normal. A search was made through all the data collected from Iceland to see if there were preferential directions with higher inclinations that were not part of the normal scatter of directions, and no positive effect was seen. Another factor shown by the Icelandic data is that the proportion of low latitude v.g.ps is very large. This was pointed out by Harrison & Watkins (1977), who showed that the pattern of v.g.ps for both eastern and western Iceland data do not not fit into the normal Fisher distribution of poles. The main reason is that the distribution has too few poles close to the mean pole and too many that are a long distance from the mean pole.

Harrison & Watkins (1977) showed that a distribution that fitted the data adequately could be derived from a Fisherian distribution for most of the poles, with the rest of the poles being randomly distributed over the surface of the Earth. The main purpose of this study by Harrison & Watkins (1977) was to compare typical continental lava flow results with those obtained from lava flows sampled by the Deep Sea Drilling Project, and so the data selection was designed to retain data from lava flows that gave a large amount of within-flow scatter. A more rigorous selection of lava flows was carried out by Dodson (1980) who found that even with the reduced number of low-latitude v.g.ps produced by this more rigorous selection, the Fisher distribution still did not adequately fit the data. Rather, he found that the bipolar distribution was a much better fit to the data (see also Onstott 1980). This distribution also has the advantage of not requiring inversion of poles in the Southern Hemisphere before a calculation is made. The reason why the bipolar distribution fits the data better is that for the same angular standard deviation the bipolar distribution reaches a maximum (i.e. the mode) at smaller angles than does the Fisher distribution, and to make up for this there is a larger proportion of poles at large distances from the mean.

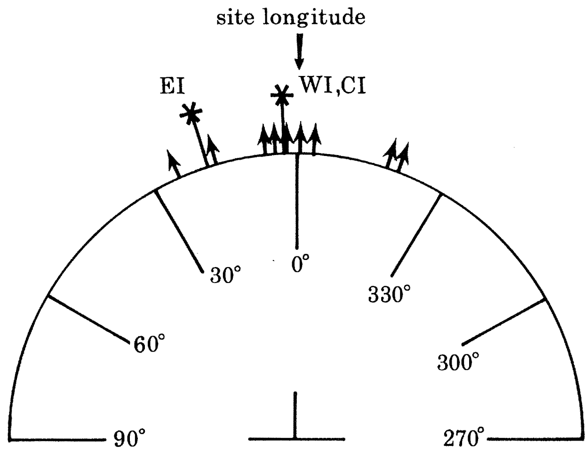

FIGURE 3. Alignment of secondary eigenvectors for Icelandic and Canary Island data, plotted relative to site longitude. The asterisk marked EI is for the eastern Iceland data set, and the asterisk marked WI, CI is for the western Iceland and Canary Island data sets. Also shown is a set of secondary eigenvectors from the staistical tests of Dodson (1980), for intermediate latitudes.

The data selection used by Dodson resulted in about one in five v.g.ps lying at latitudes within 50° of the equator. Harrison (1980) found that 17.7% of the poles lay further than 40° away from the mean pole. In a study of over 1000 lava flows from northern Iceland, Saemundsson *et al.* (1980) found that 20.0% of the poles lay within 50° of the equator. Numbers obtained from an extensive collection of lava flows in the Canary Islands show that at this lower latitude the number of v.g.ps found within 50° of the equator is only about 1 in 20, much smaller than the number for Iceland.

There is another remarkable result obtained by Dodson (1980) from his analysis of the Canary and Icelandic results. When fitting data with the bipolar distribution one obtains an eigenvector to a 3×3 matrix, which shows the mean direction. The two other eigenvectors are in a plane at 90° to the principal eigenvector and if the data are not perfectly axially symmetric, one of these secondary eigenvectors shows the meridian in which the data tend to be more clustered. Dodson found that the directions of these secondary eigenvectors for the western Iceland, eastern Iceland and Canary Islands data sets were in fact very close, too close to have been caused by chance (figure 3).

The model suggested by Dodson (1980) to explain many of the phenomena is that the non-dipole field of the Earth can be approximated by radially oriented dipoles close to the surface of the core, as first-order approximations to horizontally flowing current loops at the core surface. These radial dipoles are preferentially arranged such that there are more of them per unit area at high latitudes than at low latitudes. The preferential latitudinal arrangement was such that the probability of finding a dipole within 30° of the pole was 0.5 compared with a probability of 0.13 for a random arrangement. This model was capable of explaining the orientation of the minor eigenvectors observed in the Icelandic and Canary Island data. All of the nine intermediate latitude experiments gave intermediate eigenvectors within ±25° of the site longitude (see figure 3).

Dodson's model is also capable of explaining in a general way the fact that there are more low latitude v.g.ps from Iceland than there are from the Canary Islands. The core surface dipoles were modelled as normally distributed zero mean dipoles, which meant that there was a greater chance of producing a dipole large enough to perturb the dipole field by a large amount near Iceland than near the Canary Islands because of the greater probability of finding a dipole at the correct location. One thing that Dodson did not calculate was the relative strength of the field during times when there were low-latitude v.g.ps. This is an important consideration, in that the data from both east and west Iceland (Wilson *et al.* 1972; Harrison 1980) and another data set from Northern Iceland (Saemundsson *et al.* 1980) show that the average intensity of lava flows giving low-latitude v.g.ps is considerably less than the intensity for the other lava flows. Also, the variation of intensity with v.g.p. latitude for these data sets does not look like the data or models presented by Williams & Fuller (1981) or Hoffman (1979).

Table 2. Off-centre dipole moments

N	r/r_E	$\bar{m}$†	s.d.	r.m.s.	corrected r.m.s.	reference
8–10	0.38	—	—	0.0077–0.0193	0.0086–0.0215	1
8	0.245	0.0116	0.0878	0.0830	0.0958	2
8	0.25	0.0155	0.0875	0.0830	0.0958	2
8	0.26	0.0124	0.0813	0.0770	0.0891	2
8	0.28	0.0032	0.0620	0.0581	0.0673	2
20	0.2051	−0.0732	0.1664	0.1780	0.2019	3
34	0.2222	−0.0478	0.1895	0.1928	0.2208	3
20	0.54	0	—	0.0112	0.0847	4
20	0.54	0	—	0.0090	0.0680	4

References: 1, Lowes (1955); 2, Alldredge & Hurwitz (1964); 3, Alldredge & Stearns (1969); 4, Dodson (1980).
† Mean dipole moment, normalized by dividing by r_E^3.

Models of off-centred dipoles

There have been many models of off-centred dipoles as causes for the non-dipole field. Some of these are listed in table 2. Lowes (1955) described briefly a model which consisted of eight to ten radial dipoles placed at a radius of 0.38 (normalized to the Earth's radius), in which the dipole moments (normalized by dividing by the cube of the Earth's radius) ranged from 0.0077 to 0.0193. The four models by Alldredge & Hurwitz (1964) were attempts to model various forms of the Earth's magnetic field. They placed the dipoles between 0.245 and 0.28. The mean dipole moment is quite small because there were both positive and negative dipoles in their models. The better measurement of the strength of the dipoles is the r.m.s. value of dipole moment. The two models by Alldredge & Stearns (1969) used, in our opinion, too many dipoles. The result is that many of them are very close to neighbours, and positive and negative dipoles tend to cancel out. The result is that the r.m.s. value of the dipole moment for these models is considerably higher than the others listed in table 2. Dodson (1980) modelled the non-dipole field statistically by using 20 dipoles placed at the core–mantle boundary. The dipoles were randomly chosen from a distribution with zero mean and with a certain standard deviation, which is shown in the table.

All the models except those by Dodson (1980) placed the dipoles well within the core, the supposition being that by so doing it is possible to make a better approximation to an extended horizontal current distribution at the core–mantle boundary. We attempt to show that this is a reasonable method by calculating the potential produced by the two systems. Alldredge (1980) has given formulae for the zonal harmonics g_n^0 produced by current loops and also by vertical off-centred dipoles placed beneath the North Pole. We have calculated the variation of magnetic potential for current loops flowing at the core mantle boundary as a function of loop radius.

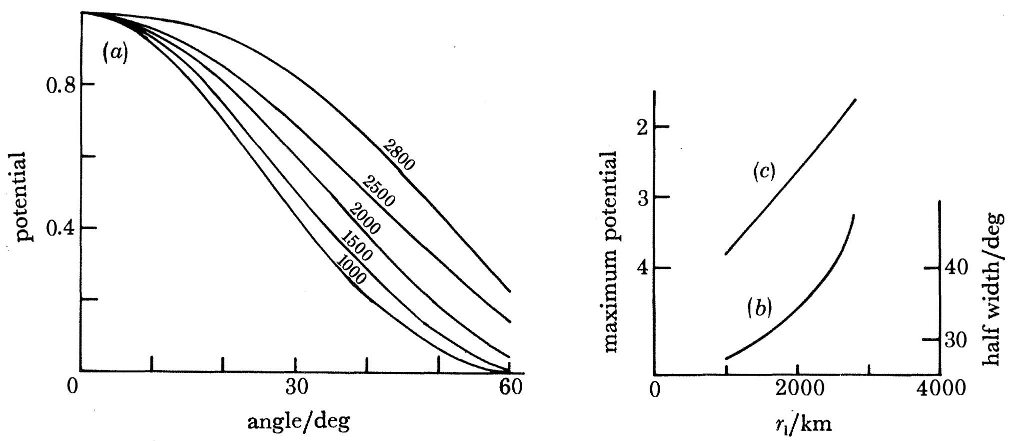

FIGURE 4. Potential from current loops. (*a*) Variation of normalized potential at Earth's surface for core surface current loops as a function of distance from axis. The radius of the loop is given in kilometres. (*b*) Half width of normalized potential as a function of radius of current loop. (*c*) Maximum potential of current loop as a function of radius of current loop.

These curves are shown in figure 4. Note that in the formulation of this problem by Alldredge (1980) the centre of the current loop was kept at a constant distance from the centre of the Earth. But one consequence of increasing the radius of a current loop flowing at the core–mantle boundary is that its centre has to get closer to the centre of the Earth. The curves on figure 4*a* show the magnetic potential as a function of radial distance from the North Pole, for current loops of radii between 1000 and 2800 km. The curves are normalized such that the maximum potential for the North Pole is unity. The curve 4*b* is the angular half width of the potential curves as a function of loop radius. As would be expected, the wider loops give potential functions of greater half width. The curve 4*c* shows the value of the potential function at its maximum (North Pole) for unit normalized magnetic moment. Again, as would be expected, if the magnetic moment is highly concentrated into a loop of small dimension, the maximum value of the potential is larger.

We have also done similar calculations for vertical off-centred dipoles, using information given by Alldredge (1980). Results are presented in figure 5. Again, it can be seen that as the dipole pproaaches the surface of the Earth (increasing value of r_d/r_E) the half width of the potential function at the Earth's surface decreases, and the maximum value of the potential increases. We have chosen to model dipoles and core surface current loops that give the same half width of the potential function. By fitting the two half-width curves in figures 4 and 5 by exponential functions, it is possible to derive an expression for the radius of the current loop as a function of the depth of the dipole. This expression is

$$R/\text{km} = 3701 - 4915 r_d/r_E. \tag{8}$$

Radii calculated for the depths given by Alldredge & Hurwitz (1964) range between 2500 and 2300 km. We can also use information presented in figures 4 and 5 to determine the dipole moment of the current loops. It turns out that the current loops have a slightly greater dipole moment than do the dipoles. This has been indicated in table 2 by showing corrected r.m.s. dipole moments, in which this correction factor is applied to the r.m.s. values for the dipoles themselves. Although the corrections are only about 15%, this is not necessarily a trivial amount, for Cox's (1968) model of field reversal calls for a reversal when the axial component of the net sum of the off-centred dipoles exceeds the axial component of the central dipole. This probability is very critically dependent on the relative strengths of the central dipole and the off-centred dipoles.

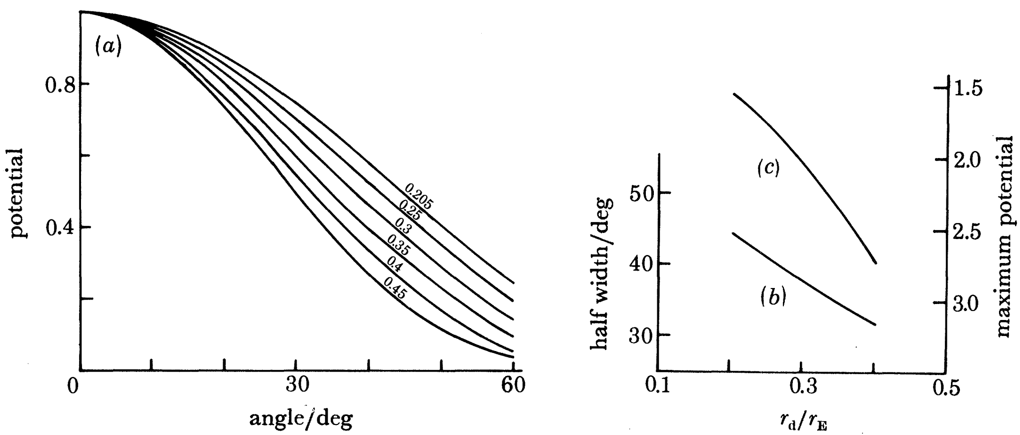

FIGURE 5. Same as figure 4, but for vertical dipoles at a distance r_d from the centre of the Earth; r_E is the radius of the Earth.

Dodson (this symposium) emphasizes that the choice of 20 radial dipoles (Dodson 1980) is somewhat arbitrary. The important parameter that controls the amount of secular variation and the number of low-latitude poles is the total variance of the radial dipoles σ_s^2, where $\sigma_s^2 = n\sigma_r^2$, n is the number of dipoles and σ_r^2 is the variance of an individual dipole. If the r.m.s. values for Dodson's dipoles are altered to be correct for eight radial dipoles, the values are 0.0177 and 0.0142. A further correction needs to be made to the dipole strengths of Dodson (1980 and this symposium) because he placed the dipoles at the core surface. If the dipole moment is adjusted for a deeper placement by using an inverse cube rule, the moments become 0.0737 and 0.0591 for dipoles placed at a radial distance of 0.26 of the Earth's radius. A further correction of 15% to give the dipole moment of a current loop brings these values up to 0.0847 and 0.0680, listed in the penultimate column of table 2.

This discussion suggests that an appropriate model for the non-dipole sources of core field consists of about eight current loops of radius about 2400 km located at the core surface, and having normalized r.m.s. dipole moments lying between 0.07 and 0.1.

Conclusions

We have shown that the use of spherical harmonics of low degree to remove the core field is not perfect. The field left after the 'core' field has been removed has sufficient power at long wavelengths to make it difficult to imagine crustal sources with enough magnetization to produce the desired field variation. Instead we suggest that off-centred dipoles located within the core are a better way to describe the non-dipole field of the Earth. We believe that it is possible to remove long-wavelength signals more completely by using off-centred dipoles buried deep within the core than by using spherical harmonics. These dipoles represent core surface current circulation systems. Data from Iceland and the Canary Islands show that viable models of the Earth's non-dipole field can be produced that explain many of the field variations recorded in extensive lava flow piles in these two islands.

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References (Harrison & Carle)

Alldredge, L. D. 1980 *J. Geomagn. Geoelect., Kyoto* **32**, 357–364.
Alldredge, L. D. 1981 *J. geophys. Res.* **86**, 3021–3026.
Alldredge, L. D. & Hurwitz, L. 1964 *J. geophys. Res.* **69**, 2631–2640.
Alldredge, L. D. & Stearns, C. O. 1969 *J. geophys. Res.* **74**, 6583–6593.
Bullard, E. C. 1967 *Earth planet. Sci. Lett.* **2**, 293–300.
Carle, H. M. & Harrison, C. G. A. 1982 *Geophys. Res. Lett.* **9**, 265–268.
Chapman, S. & Bartels, J. 1940 *Geomagnetism.* Oxford: Clarendon Press.
Cox, A. 1968 *J. geophys. Res.* **73**, 3247–3260.
Cox, A. 1975 *Rev. Geophys. Space Phys.* **13**, 33–51.
Dodson, R. E. 1980 *J. geophys. Res.* **85**, 3606–3622.
Harrison, C. G. A. 1976 *Geophys. Jl R. astr. Soc.* **47**, 257–283.
Harrison, C. G. A. 1980 *J. geophys. Res.* **85**, 3511–3522.
Harrison, C. G. A. 1981 In *The sea*, vol. 7 (ed. C. Emiliani), pp. 219–239. New York: Wiley.
Harrison, C. G. A. & Watkins, N. D. 1977 *J. geophys. Res.* **82**, 4869–4877.
Harrison, C. G. A. & Watkins, N. D. 1979 *J. geophys. Res.* **84**, 627–635.
Hoffman, K. A. 1979 *Earth planet. Sci. Lett.* **44**, 7–17.
Langel, R. A. & Estes, R. H. 1982 *Geophys. Res. Lett.* **9**, 250–253.
Lowes, F. J. 1955 *Ann. Geophys.* **11**, 91–94.
Lowes, F. J. 1966 *J. geophys. Res.* **71**, 2179.
Onstott, T. C. 1980 *J. geophys. Res.* **85**, 1500–1510.
Saemundsson, K., Kristjansson, L., McDougall, I. & Watkins, N. D. 1980 *J. geophys. Res.* **85**, 3628–3646.
Williams, I. & Fuller, M. D. 1981 *J. geophys. Res.* **86**, 11657–11665.
Wilson, R. L. & Ade-Hall, J. M. 1970 In *Palaeogeophysics* (ed. S. K. Runcorn), pp. 307–312. New York: Academic Press.
Wilson, R. L., Dagley, P. & McCormack, A. G. 1972 *Geophys. Jl R. astr. Soc.* **28**, 213–224.
Zmuda, A. J. 1958 *J. geophys. Res.* **63**, 447–490.

Discussion

K. M. Creer (*Department of Geophysics, University of Edinburgh, U.K.*). Models such as that of Alldredge & Hurwitz (1964) in which the geomagnetic field is fitted by several dipoles located in the outer core plus a central dipole assume the field to be static. That is to say the core is assumed to be transparent so that fields produced by sources (whether they be dipoles or current loops) in the far side of the core from the observer are transmitted across the core without attenuation. In fact, the geomagnetic field is not static so the assumption of no attenuation is not correct. So my question is: In the models that Dr Harrison has described, has he allowed for attenuation of the signal? If not, can he suggest how this might be done, bearing in mind the fact that the outer core is not solid?

C. G. A. Harrison. We have not allowed for attenuation of the signal. The sources are believed to be near the core–mantle boundary because deeper time-varying sources would be shielded by the conducting core from having an effect at the Earth's surface. This suggests that one possible method of treating this problem would be to disregard the effect of any individual dipole at locations greater than 90° away from the dipole (i.e. in the opposite hemisphere to the dipole).

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